Diminishing marginal utility implies that losses hurt more than gains

Basil Halperin

@basilhalperin

Assume:

$$U : \mathbb{R} \to \mathbb{R} \tag{1}$$

$$U' > 0 \tag{2}$$

$$U'' < 0 \tag{3}$$

Further assume U is smooth enough that Taylor's theorem holds.

Let $\varepsilon > 0$
By Taylor expansion with Lagrange error term,

$$U(W) = U(W_0) + U'(W_0)(W - W_0) + R(W) \tag{4}$$

$$R(W) = \frac{1}{2}U''(c)(W - c)^2, \text{ some } c \tag{5}$$

Want to show: $U(W_0) - U(W_0 - \varepsilon) > U(W_0 + \varepsilon) - U(W_0)$
$\iff 2U(W_0) > U(W_0 + \varepsilon) + U(W_0 - \varepsilon)$
$\iff 2U(W_0) > [U(W_0) + U'(W_0)(W_0 + \varepsilon - W_0) + R(W_0 + \varepsilon)] + [U(W_0) + U'(W_0)(W_0 - \varepsilon - W_0) + R(W_0 - \varepsilon)$ by (4)
$\iff 0 > R(W_0 + \varepsilon) + R(W_0 - \varepsilon)$

This is true, by (5), as $U'' < 0$ by (3) $\square$