When Does Automating AI Research Produce Explosive Growth?

Tom Davidson [1] **Basil Halperin** [2] **Thomas Houlden** [3] **Anton Korinek** [4]

[1]Forethought, [2]University of Virginia, [3]Columbia University, [4]University of Virginia

December 2025

AI companies are aiming to automate AI research

> “ Advanced AI is interesting for many reasons, but perhaps **nothing is quite as significant as the fact that we can use it to do faster AI research.”**
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> “ We have set **internal goals** of having an automated AI research intern by September of 2026 … and a **true automated AI researcher by March of 2028**.”
>
> *Sam Altman*, Oct 2025

The software-hardware model of AI

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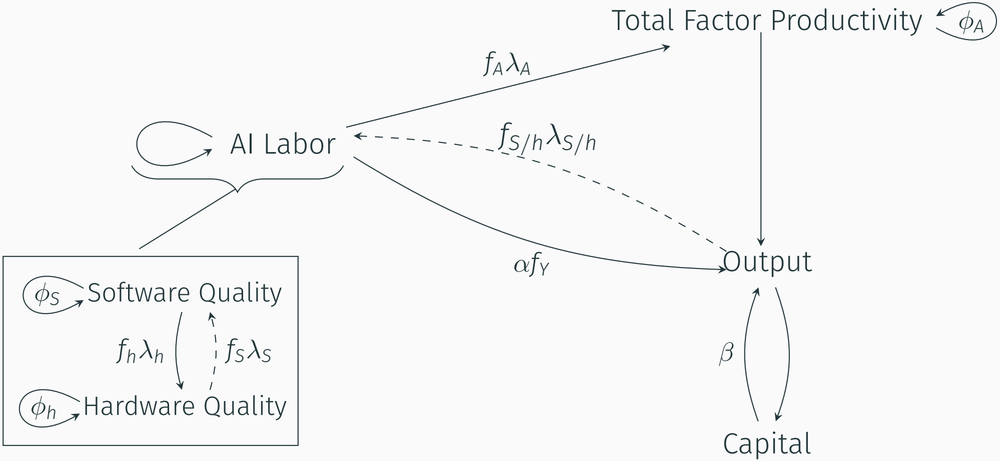

Roadmap ‣ Lit review

1. **Building blocks of the model**

   Lesson 1: diminishing returns prevent growth explosions

   Lesson 2: technological feedback loops create spillovers across sectors, offsetting diminishing returns

   Lesson 3: *economic* feedback loops create spillovers across sectors, offsetting diminishing returns

   Lesson 4: automation offsets diminishing returns

   Lesson 6 (empirical): diminishing returns are low in software/hardware

2. **The software-hardware model**
3. **Scope of claims**

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The intelligence explosion

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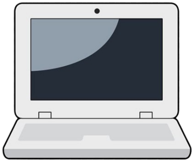

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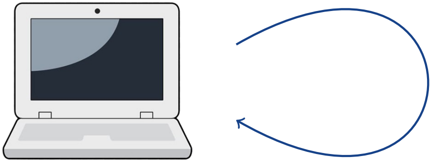

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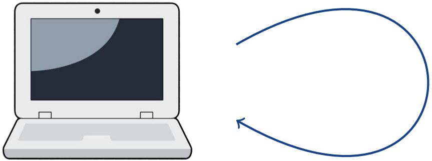

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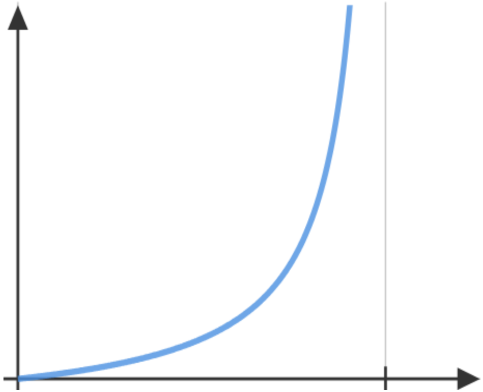
$$\dot{S}_t = S_t^{1+\phi}$$

$$\phi > 0$$



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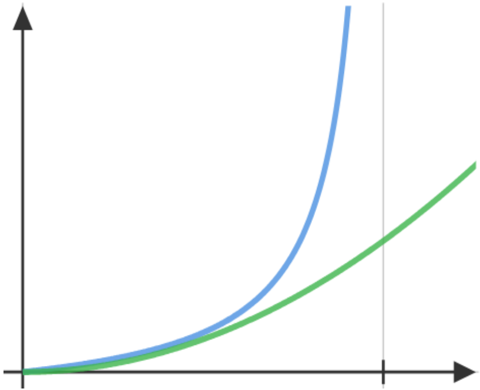
$$\dot{S}_t = S_t^{1+\phi}$$

$$\phi \in (-1, 0)$$



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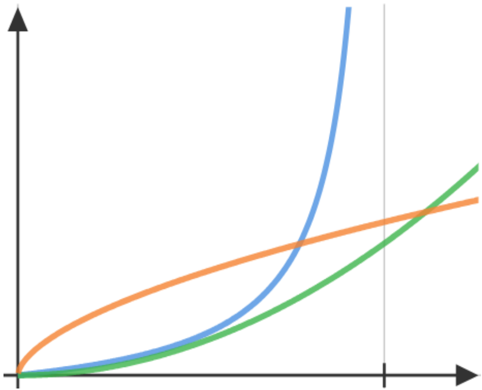

The intelligence explosion? The role of diminishing returns

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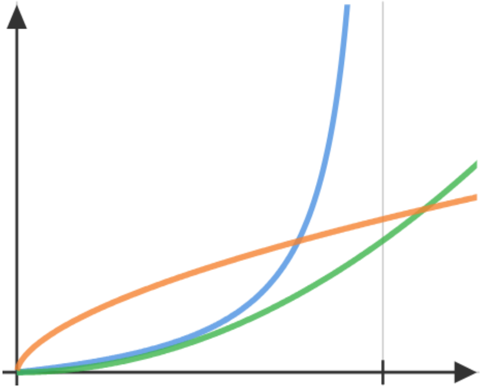

Will an *intelligence* explosion cause an *economic* explosion?

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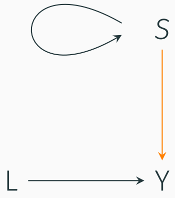

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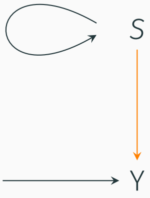

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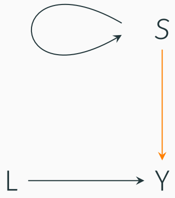

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- **Economic singularity** condition:

$$\phi > 0$$

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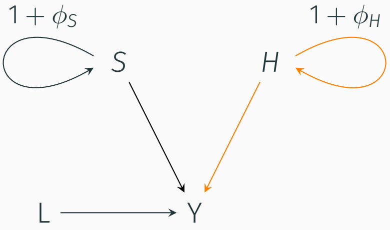
A networked semi-endogenous growth model

$$\dot{S}_t = S_t^{1+\phi_S}$$
$$\dot{H}_t = H_t^{1+\phi_H}$$
$$Y_t = (S_t H_t)^{1/2} L_t^{\alpha}$$



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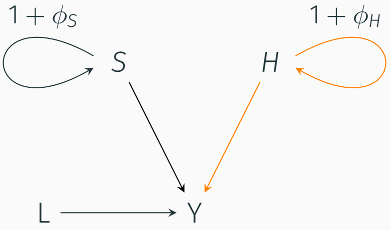

Economic singularity condition:

$$\phi_S \text{ or } \phi_H > 0$$

A networked semi-endogenous growth model: technological spillovers

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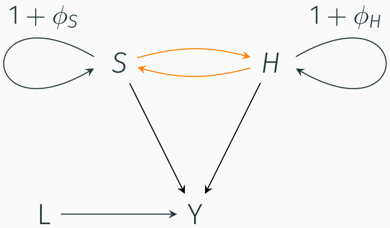

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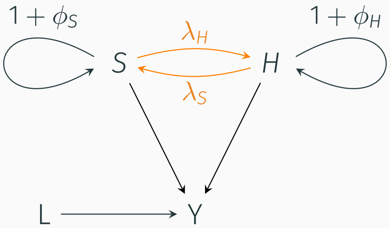

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$$\underbrace{\qquad\qquad\qquad}_{\text{direct effects}} + \underbrace{\qquad}_{\text{indirect effects}} > 1$$

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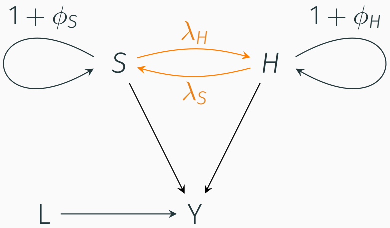

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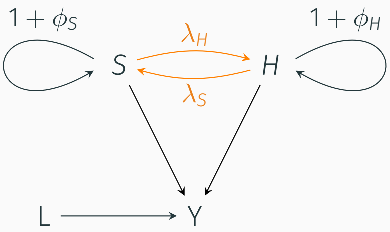

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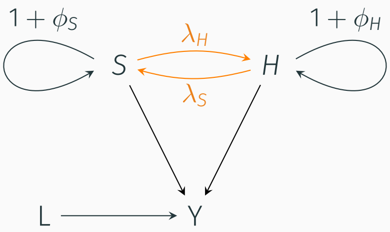

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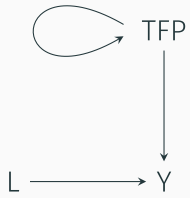

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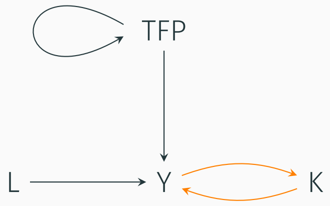

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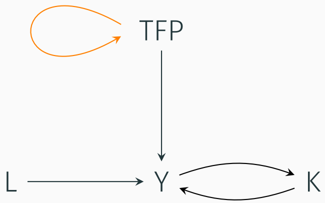

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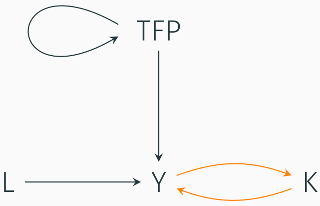

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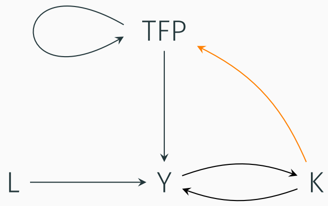

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Focusing on accumulable factors:

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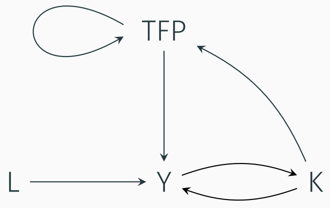

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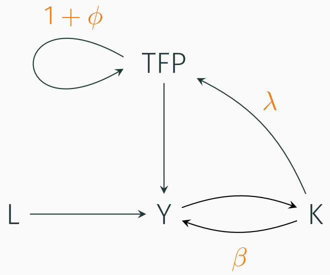

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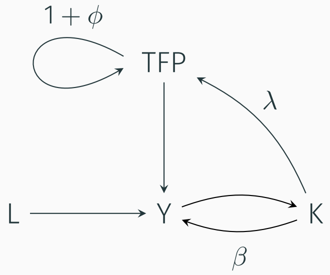

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$$\phi > 0 \text{ or } \beta > 1$$

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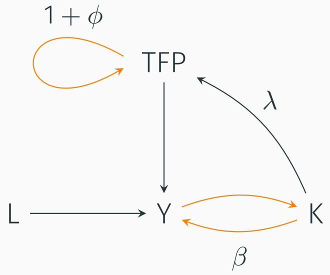

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Focusing on accumulable factors:

$$\dot{A}_t = \text{stuff} \cdot A_t^{1+\phi} K_t^\lambda$$
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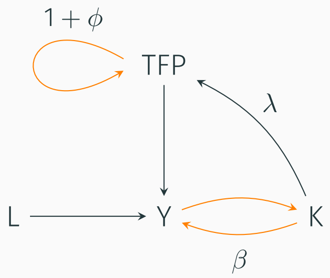
Proposition (explosive systems).
System explodes in finite time if the exponent matrix, $\begin{bmatrix} 1+\phi & \lambda \\ 1 & \beta \end{bmatrix}$, has an eigenvalue > 1.



Explosion conditions:

$$\phi > 0 \text{ or } \beta > 1$$
$$\underbrace{(1+\phi)+\beta}_{\text{direct effects}} - (1+\phi)\beta > 1$$

Accumulable factors **create** economic feedback loops

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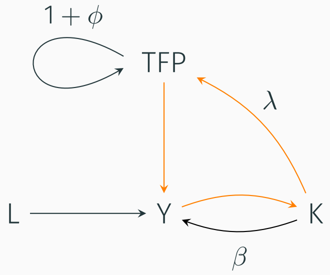

Explosion conditions:

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$$\underbrace{(1+\phi)+\beta}_{\text{direct effects}} - (1+\phi)\beta + \underbrace{\lambda \cdot 1}_{\text{indirect effects}} > 1$$

The canonical semi-endogenous growth model

$$\dot{A}_t = A_t^{1+\phi}(\ell L_t)^{\lambda}(\kappa K_t)^{\lambda}$$

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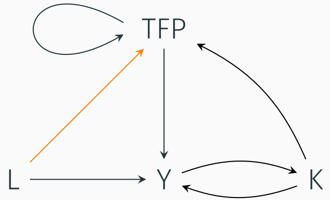

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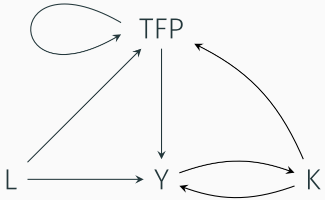

Best guess calibration:

- $\phi = -3.4$ (Bloom et al 2020) — $\phi > 0$ ✗
- $\beta = 0.4$ (capital share in production) — $\beta > 1$ ✗
- $\lambda = 0.1$ (capital share in R&D) — $(1+\phi)+\beta-(1+\phi)\beta+\lambda > 1$ ✗

1. Building blocks of the model

Lesson 1: diminishing returns prevent growth explosions

Lesson 2: technological feedback loops create spillovers across sectors, offsetting diminishing returns

Lesson 3: *economic* feedback loops create spillovers across sectors, offsetting diminishing returns

Lesson 4: automation offsets diminishing returns

Lesson 6 (empirical): diminishing returns are low in software/hardware

2. The software-hardware model

3. Scope of claims

$$\dot{A}_t = A_t^{1+\phi}(\ell_A L_t)^\lambda(\kappa_A K_t)^\lambda$$
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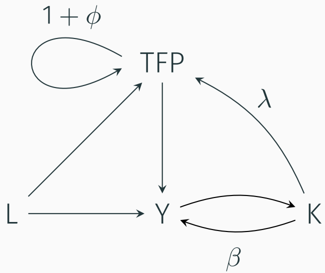

Explosion conditions:

$$\phi > 0$$
$$\beta > 1$$
$$(1+\phi) + \beta - (1+\phi)\beta + \lambda > 1$$

Introducing automation: offsets diminishing returns

[Aghion, Jones, and Jones (2019)]

$$\dot{A}_t = A_t^{1+\phi}(\ell_A L_t)^\lambda(\kappa_A K_t)^\lambda$$

$$Y_t = A_t\,(\ell_Y L_t)^{\alpha(1-f_Y)}\,(\kappa_Y K_t)^{\beta+f_Y\alpha}$$

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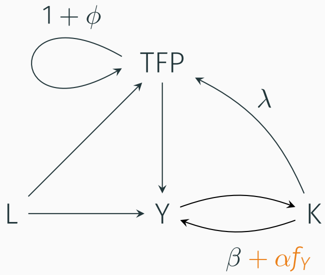

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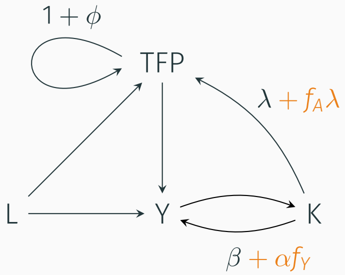

Explosion conditions:

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CHM25 policy

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Software-hardware model: overview

Canonical semi-endogenous growth model, plus:

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Canonical semi-endogenous growth model, plus:

1. Automation of labor with “AI”
2. AI = software · hardware · hardware quality

The software-hardware model of AI

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The software-hardware model of AI

AI substituting for labor:

$$\text{AI} \equiv Z = \underbrace{S}_{\text{software}} \cdot \underbrace{C}_{\text{hardware}} = \underbrace{S}_{\text{software}} \cdot \underbrace{c \cdot h}_{\text{hardware}}$$

- **Software:** “algorithmic efficiency”
- **Hardware:** computer hardware (“compute”)
  - Hardware quantity: c, “number of computer chips”
  - Hardware quality: h, “how many calculations (FLOPs) per chip”

Software and hardware evolution

Hardware accumulates: just another form of capital

$$\dot{C}_t = s_C Y_t - \delta_C C_t$$

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Remember ideas production function:

$$\dot{A}_t = (\ell_A L_t)^{\lambda_A} A_t^{1+\phi_A}$$

Software and hardware evolution

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Hardware quality is like ideas and investment-specific technical change: better hardware quality allows for *faster accumulation of effective hardware*

[a la Greenwood-Hercowitz-Krusell]

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Effective labor in sector X: *(with automation)*

$$\hat{L}_{x,t} = (\ell_x L_t)^{1-f_x} \cdot Z_{x,t}{}^{f_x}$$

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$$= (\ell_x L_t)^{1-f_x} \cdot \left(\underbrace{S_t}_{\text{software}} \cdot \underbrace{c_{x,t} \cdot h_t}_{\text{hardware}} \right)^{f_x}$$

Note: effective labor accumulates

The software-hardware model: equations

Output: $Y_t = A_t \hat{L}_{Y,t}^{\alpha} K_t^{\beta}$

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Ideas: $$\dot{A}_t = \hat{L}_{A,t}^{\lambda_A} A_t^{1+\phi_A}$$

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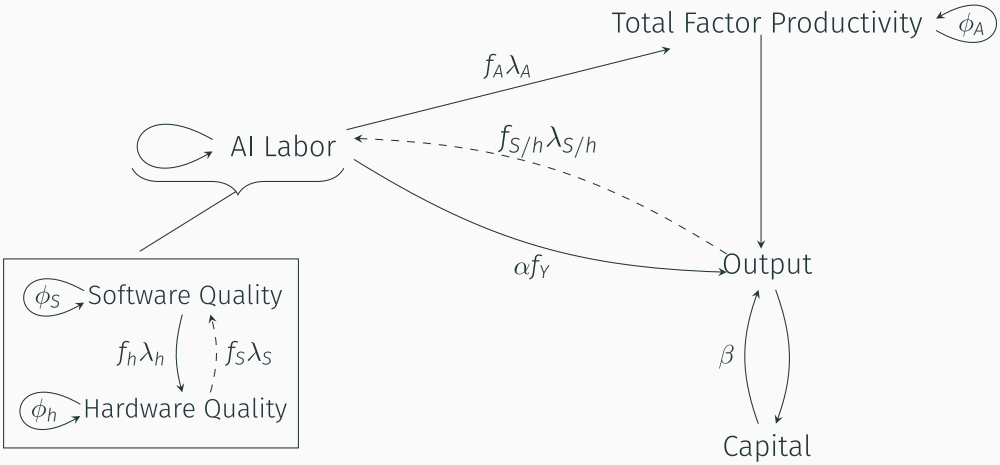
AI automation: $$\hat{L}_{x,t} = L_{x,t}^{1-f_x} \cdot (S_t \cdot c_{x,t} \cdot h_t)^{f_x}$$

The software-hardware model: diagram



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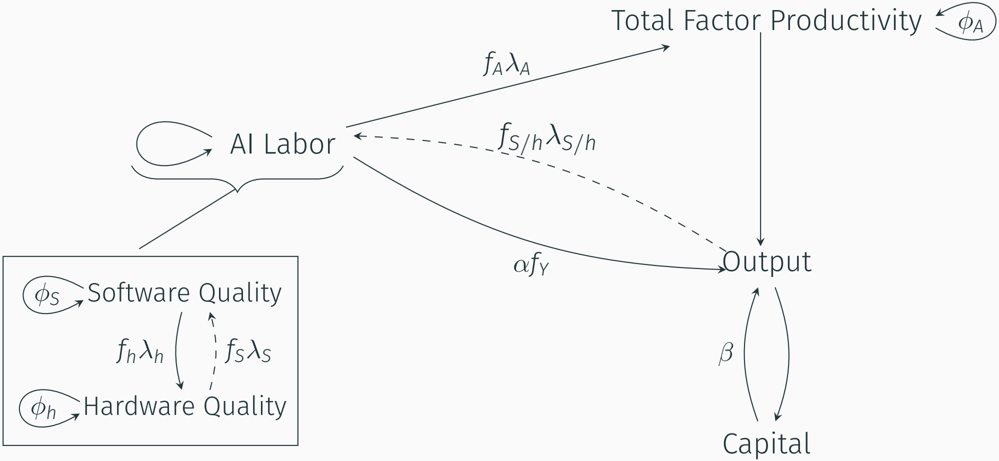

Strength of feedback increasing with all exponents

Explosion condition

Simplify the problem by assuming complete depreciation. Substituting in effective labor expressions and removing non-accumulable factors

$$\dot{S}_t \propto S_t^{f_S \lambda_S \frac{1-\beta}{1-f_Y\alpha-\beta}+1+\phi_S} h_t^{f_S \lambda_S \frac{1-\beta}{1-f_Y\alpha-\beta}} A_t^{\frac{f_S \lambda_S}{1-f_Y\alpha-\beta}}$$

$$\dot{h}_t \propto S_t^{f_h \lambda_h \frac{1-\beta}{1-f_Y\alpha-\beta}} h_t^{f_h \lambda_h \frac{1-\beta}{1-f_Y\alpha-\beta}+1+\phi_h} A_t^{\frac{f_h \lambda_h}{1-f_Y\alpha-\beta}}$$

$$\dot{A}_t \propto S_t^{f_A \lambda_A \frac{1-\beta}{1-f_Y\alpha-\beta}} h_t^{f_A \lambda_A \frac{1-\beta}{1-f_Y\alpha-\beta}} A_t^{\frac{f_A \lambda_A}{1-f_Y\alpha-\beta}+1+\phi_A}$$

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Applying explosion proposition yields **explosion threshold**:

$$\frac{1}{1-\beta} f_A r_A + \frac{\alpha}{1-\beta} f_Y + f_S r_S + f_h r_h > 1$$

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r **factor:** for $x \in \{A, S, h\}$,

$$r_x \equiv \frac{\lambda_x}{-\phi_x}$$

- Intuition: in canonical model, $g_A = r_A \cdot$ population growth

Calibrating parameters

Explosion condition: $\frac{1}{1-\beta} f_A r_A + \frac{\alpha}{1-\beta} f_Y + f_S r_S + f_h r_h > 1$

Term	Parameter	Estimate	Source
Labor share	$\alpha \equiv 1-\beta$	0.6	–

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Interpretation: Software and hardware have **much lower** diminishing returns to research than the rest of the economy $\implies$ if software/hardware grow as share of economy, large growth effects

1. **Building blocks of the model**

   Lesson 1: diminishing returns prevent growth explosions

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2. **The software-hardware model**

3. **Scope of claims**

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4. **Endogenous automation**
5. **More:**
   - Endogenous allocation rules
   - Decentralized allocation: roles of industrial organization + externalities
   - Learning by doing
   - Capital adjustment costs
   - Time to build

How much automation is necessary for a growth explosion?

CHM25 policy

Automated Factors	Automation Threshold $2 \times g_Y^{BGP}$
S	–
H	–
S, H	–
$H,\ Y$	12%
$S,\ Y$	16%
$S,\ H,\ Y$	11%
$S,\ H,\ A,\ Y$	8%
$S,\ H,\ A,\ Y$ (10% fixed factor)	9%

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▸ CHM25 ▸ policy

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▸ CHM25 ▸ policy

Automated Factors	Automation Threshold	
	$2 \times g_Y^{BGP}$	$\infty \times g^{BGP}$ (Hyperbolic)
S	–	$\sim 100\%$
H	–	20%
S, H	–	17%
$H,\ Y$	12%	17%
$S,\ Y$	16%	50%
$S,\ H,\ Y$	11%	14%
$S,\ H,\ A,\ Y$	8%	13%
$S,\ H,\ A,\ Y$ (10% fixed factor)	9%	14%

Conclusion: "Why wasn't automating agriculture enough for a growth explosion?"

Tractors $\rightarrow$ more food $\rightarrow$ more people $\rightarrow$ better tractors $\rightarrow \cdots$

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Conclusion: "Why wasn't automating agriculture enough for a growth explosion?"

Tractors → more food → more people → better tractors → · · ·

1. Maybe it was? Our condition speaks to 'are we *on track*' for a growth explosion

2. 'Diminishing returns' is one reason; diminishing returns are *less* strong in hardware and software

Conclusion: "Why wasn't automating agriculture enough for a growth explosion?"

Tractors $\rightarrow$ more food $\rightarrow$ more people $\rightarrow$ better tractors $\rightarrow \cdots$

1. Maybe it was? Our condition speaks to 'are we *on track*' for a growth explosion

2. 'Diminishing returns' is one reason; diminishing returns are *less* strong in hardware and software

3. Bottlenecks or other limits: we do not speak to *all* limits

Thank you!

Appendix

On bottlenecks

Cobb-Douglas: with $\alpha > 0$

$$Y = L^{\alpha} K^{1-\alpha}$$

Fix L, send $K \to \infty \implies Y \to \infty$.

Potential bottlenecks:

- **Compute** bottlenecking algorithmic progress
- **Algorithmic progress** bottlenecking compute
- **Energy** bottlenecking everything
- **Data** bottlenecking everything

CES with complements: with $\phi < 0$

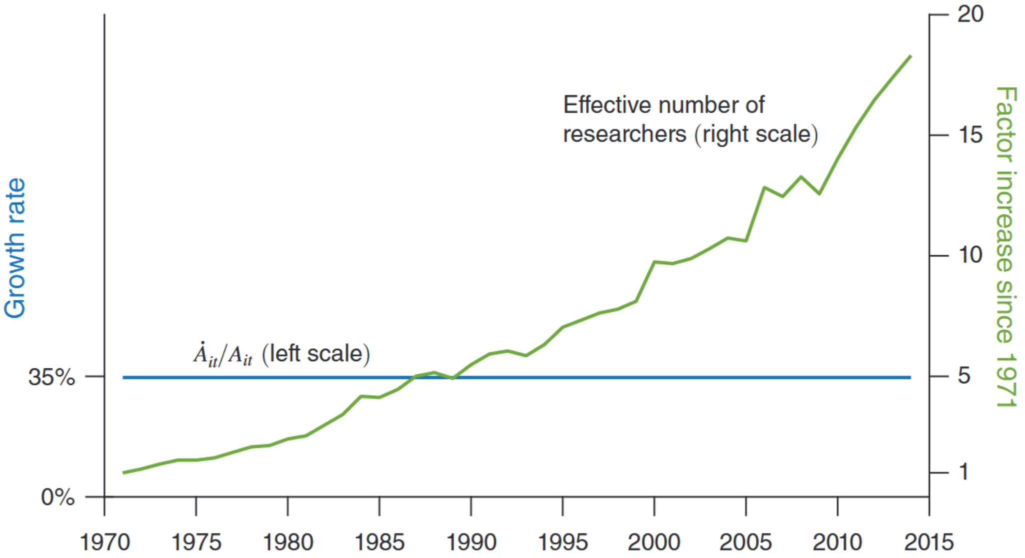
$$Y = \left[L^{\phi} + K^{\phi}\right]^{1/\phi}$$

Fix L, send $K \to \infty \implies Y = L$

Potential reasons to think bottlenecks will be less of an issue:

- 2x efficient algorithims $\implies$ 2x as many experiments
- Aum and Shin (2024): software and labor are substitutes not complements

Could ϕ be falling over time? Doesn't appear to be for Moore's Law



Multisector semi-endogenous growth model

Standard one-sector model:

- Idea production functions:

$$\dot{A}_t = L_t^{\lambda} A_t^{1+\phi}$$

- BGP:

$$\frac{\dot{A}}{A} = \frac{\lambda}{-\phi} n$$

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Two-sector model:

- Aggregate TFP: $A_t = A_{1t}^{\sigma_1} A_{2t}^{\sigma_2}$
- Idea production functions:*

$$\dot{A}_{it} = (s_i L_t)^{\lambda_i} A_{it}^{1+\phi_i}$$

* s_i exogenous and constant ("Solow-style"). It can be shown, though, that optimally s_1/s_2 is constant under Cobb-Douglas aggregation.

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Comparative static: Suppose $-\phi_1 > -\phi_2$. Increase σ_2. Obviously $g_A \uparrow$

---

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