# The ZLB is NBD: 5 theses on the New Keynesian "liquidity trap"

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**Abstract**

I make five conceptual points about optimal monetary and fiscal policy at the zero lower bound (ZLB) in representative agent New Keynesian models, using the simplest possible version of such a model.

(1) Monetary policy is typically described as facing a time consistency problem at the zero lower bound; but if ZLB episodes are a *repeated* game rather than a one-shot game – as is empirically realistic – then the time consistency problem can be easily overcome by reputational effects.

(2) The ZLB is not special, in terms of the constraint it creates for monetary policy: an *intra*temporal rigidity, such as the minimum wage or rent control, creates exactly the same kind of constraint on monetary policy as the *inter*temporal rigidity of the ZLB.

(3) Austerity is stimulus: in the representative agent New Keynesian model, fiscal stimulus works through the *change* in government spending. Promising to cut future spending – committing to austerity – has precisely the same effect on inflation and the output gap as a decision to raise spending today.

(4) Fiscal stimulus can be contractionary, when targeted heterogeneously: if fiscal spending is targeted at certain sectors, this can in fact lower inflation and deepen the output gap.

(5) Fiscal policy faces a time consistency problem at the ZLB, just as monetary policy does.

Overall, I suggest that – in this class of models – the power of monetary policy at the ZLB has been underrated, and the power of fiscal policy has been overrated.

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# 1 Introduction

This paper reevaluates optimal monetary and fiscal policy at the zero lower bound (ZLB) in the workhorse representative agent New Keynesian (RANK) model. I write down a simple and transparent two-period version of the RANK model as a baseline, which I extend minimally as the paper develops, to highlight a set of five underappreciated or novel implications of the model for optimal monetary and fiscal policy. The unifying theme is that monetary policy is less constrained at the ZLB than is commonly thought, whereas fiscal policy in turn is more constrained than typically thought.

I begin with the canonical infinite-horizon New Keynesian model, as described in the Woodford (2003) or Gali (2008) textbooks. I transform the model into a simple two-period model by assuming that after the first two periods, prices are completely flexible. This technique, as in Krugman (1998), creates a "simplest possible version of the model" that is usefully transparent for presenting the conceptual points in the rest of the paper.

This setup is useful for highlighting a baseline result – well-known in the literature since the seminal work of Krugman (1998), if often misunderstood in popular discourse – that at the ZLB, monetary policy is not "pushing on a string". Indeed, as long as the ZLB is only temporarily binding, a credible central bank can achieve any desired level of inflation under standard assumptions. The typical concern, however, is that doing so requires the central bank to commit to expansionary monetary policy *after* the ZLB has ceased to bind, and that this commitment to future expansionary policy is not a time-consistent promise (Eggertsson and Woodford 2003). In the memorable language of Krugman (1998), it is difficult for central banks to "credibly promise to be irresponsible." In this view, the ZLB is less of a "liquidity trap" than it is an "*expectations* trap".

I refer to the model just described as the "baseline model" for the rest of the paper, and I offer a sequence of extensions to it to illustrate my five main points.

**Repeated ZLB episodes.** The first proposition of the paper, also shown in the important work of Nakata (2018), is that if ZLB episodes are a repeated game – rather than a one-time event, as is typically modeled – then the promises of a ZLB-constrained central bank can easily be credible. During any given single ZLB episode, the central bank recognizes that if it deviates from its promised commitment, this will harm its reputation and credibility for *future* promises during *future* ZLB episodes. If ZLB episodes are sufficiently frequent, then the reputational cost will outweigh the gains of reneging on the promise, and the central bank will choose to honor its promise.

The intuition for this result comes directly from the result in basic game theory that actions which are not credible in one-shot games may easily be credible in repeated games. I write down a simple modification of the baseline model to illustrate and make precise this claim, and I show how it is merely the deflationary mirror image of the Barro and Gordon (1982) critique that reputation can help central banks overcome the *inflationary* time consistency problem of Kydland and Prescott (1976). To paraphrase Krugman (1998): it is easy to credibly promise to be 'irresponsible' *if* your reputation is on the line.

**Intertemporal vs. intratemporal distortions.** Even if central banks can overcome the time consistency problem at the ZLB, they still cannot achieve the first best in the New Keynesian model – which absent the ZLB they could do – because the ZLB constraint means that there are fewer instruments than targets. The second proposition of the paper is to show and make precise the idea that this limitation is not special: the ZLB is qualitatively no different from any other exogenous nominal rigidity.

The real interest rate is the relative price of consumption today versus consumption tomorrow, and the zero lower bound is merely a nominal price floor on that relative price; a nominal price floor (or any nominal rigidity) on *any* relative price between *any* two goods would constrain monetary policy in the same way as the ZLB.

This point is typically obscured by the fact that the baseline RANK model works with a single, representative consumption good. I modify the baseline model to include two consumption goods at any given point in time, and I show that the *inter*temporal nominal rigidity of the ZLB constrains monetary policy in a way that is isomorphic to an *intra*temporal nominal rigidity on the relative price of the two consumption goods. There are countless examples of such heterogeneous intratemporal rigidities in reality. The restrictions imposed by minimum wage laws are merely one example (Minton and Wheaton 2021).

**Austerity is stimulus.** Turning to fiscal policy, the third result of the paper is to show that the welfare-relevant effects of fiscal stimulus in the baseline RANK model depend exclusively on the *change* in government spending, rather than the level. That is, government spending in RANK only affects inflation and the output gap through *the expected growth in government spending from today to tomorrow.* As a result, austerity is stimulus: the promise of spending cuts tomorrow is isomorphic to temporarily raising spending today, in terms of the effect on inflation and the output gap. I show this by extending the baseline model to include government purchases of and production of public goods (as in Werning 2012, Woodford 2011, and Eggertsson 2001).

The intuition for this result comes from consumption smoothing: if the government is going to buy fewer goods tomorrow than today, then tomorrow there will be more goods available for private consumption than today. Under flexible prices, households would therefore want to smooth their consumption over time and borrow to consume more today, in order to match their higher expected consumption tomorrow more closely. This demand for borrowing causes the real rate to rise in flexible price models – or in sticky price models, causes the *natural* rate of interest to rise. Given a nominal interest rate fixed at the zero lower bound, the rise in the natural real rate is stimulative, as it reduces the gap between the policy rate and the natural rate.

A second piece of intuition for this result is that the mechanism through which fiscal stimulus operates is perfectly isomorphic (in terms of the effect on inflation and the output gap) to the stimulative effect of a temporary negative productivity shock at the ZLB. It is well-known that temporary negative productivity shocks are expansionary at the ZLB.[1] Both negative productivity shocks and positive fiscal stimulus lower current consumption relative to future consumption, pushing up the natural rate, which is stimulative at the ZLB. In short, government spending in RANK is – in terms of its effects on the output gap and inflation – a negative productivity shock to the economy.

**Contractionary stimulus.** This analogy to negative productivity shocks points to the fourth result: if positive fiscal stimulus is targeted at certain specific sectors rather than spread evenly throughout the economy, then it can in fact be *contractionary.* For example, if fiscal stimulus consists solely of increased purchases in the goods sector – rather than being spread evenly across every sector, from goods to agriculture to services and beyond – then this may cause deflation and create an output gap.

The intuition for this comes from the work of Guerrieri, Lorenzoni, Straub, and Werning (2022), who show that while a negative productivity shock in a *one-sector* model always raises the natural rate, it may lower the natural rate in a *multisector* model when the shock doesn't affect all sectors equally. As noted above, positive fiscal stimulus in RANK is isomorphic to a negative productivity shock. Thus, analogously to the result of Guerrieri-Lorenzi-Straub-Werning, while increased government purchases of a single representative good are always expansionary at the ZLB, government purchases which are heterogeneous across multiple sectors may be contractionary by lowering the natural rate. I demonstrate this in a multisector extension to the baseline model.[2]

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[1] The benefits of negative productivity shocks at the ZLB are discussed, among many other papers, in Eggertsson (2011, 2012). Wieland (2019) as well as Garín, Lester, and Sims (2019) empirically evaluate this prediction unfavorably. Kiley (2016) and Eggertsson and Garga (2019) discuss this issue under the sticky information friction (Mankiw and Reis 2002) versus the Calvo-Yun sticky price friction.

[2] Aoki (2001), Benigno (2004), Woodford (2003, ch. 6 sec. 4.3), and Rubbo (2022) study optimal monetary policy in multisector New Keynesian models away from the ZLB. My analysis on this issue differs by studying optimal fiscal

**Time inconsistency of fiscal policy.** The emphasis above on fiscal stimulus as inherently *intertemporal* brings us to the fifth and final result of the paper: optimal fiscal policy faces a time consistency problem at the zero lower bound. To my knowledge, this is the first paper to analytically characterize the time consistency of optimal fiscal policy at the ZLB.

The intuition for this time inconsistency is that optimal fiscal policy would like to spread changes in government consumption over time, for reasons of optimal provision of public goods. As a consequence, optimal fiscal stimulus – which, recall, is driven by the *change* in spending – involves *both* an increase in fiscal spending during the ZLB *and* a cut in spending after the ZLB, rather than simply a large increase during the ZLB. Even a fully beneficent fiscal authority has an incentive to deviate from this policy after the ZLB and to not follow through on the promised austerity, because the austerity is distortionary for public goods provision. Of course, fiscal policy may overcome its time consistency problem in the same way as monetary policy – through a reputational mechanism – though plausibly this is more challenging for fiscal authorities who operate under relatively more severe political economy constraints.

**Layout.** The literature on optimal monetary and fiscal policy in the New Keynesian liquidity trap is large, and rather than give a literature review I cite papers throughout where they are relevant. The rest of the paper proceeds directly as it was summarized above. In section 2, I introduce the baseline model and review the idea that, in modern models, monetary policy is able to affect inflation even at the ZLB. Sections 3 and 4 advance the two theses on optimal monetary policy; sections 5 through 7 advance the three theses on optimal fiscal policy. I conclude with some remarks on the limitations of the representative agent New Keynesian model.

# 2 Baseline model

In this section, I set up a minimalistic New Keynesian model of the ZLB. It will serve as the baseline model for the rest of the paper, as we extend it in various directions and use it to conduct policy experiments. Readers familiar with Eggertsson and Woodford (2003) or Werning (2012) may wish to look at equations (5)-(12), the discussion in section 2.3, and to skip to section 3.

## 2.1 Setup

policy, and (crucially) by allowing for intra-sector and inter-sector elasticities of substitution to differ, as in Guerrieri, Lorenzoni, Straub, and Werning (2022).

I start directly from the canonical log-linearized New Keynesian model that results from the basic setup with Calvo pricing and no capital (c.f. Gali 2008 or Bergholt 2012, for example).[3]

The core equations are the New Keynesian Phillips Curve (NKPC),

$$\pi_t = kx_t + \beta E_t \pi_{t+1} \tag{1}$$

and the Euler equation (EE) written in terms of the output gap,

$$x_t = E_t x_{t+1} - \sigma[i_t - E_t \pi_{t+1} - r_t^n] \tag{2}$$

The notation is standard: $\pi_t$ is inflation; $x_t$ is the output gap; $k$ is a constant that depends on preference and technological parameters; $\beta$ is the steady state rate of time preference; $\sigma$ is the inverse of the intertemporal elasticity of substitution; $i_t$ is the (gross) nominal interest rate.

$r_t^n$ is the (gross) natural interest rate, which in the baseline framework only depends on the exogenous rate of time preference $\rho_t$:[4]

$$r_t^n = \rho_t \tag{3}$$

The central bank's per-period welfare loss function is (to a second-order approximation):

$$\mathbb{W}_t = \pi_t^2 + \lambda x_t^2 \tag{4}$$

Here, $\lambda$ is a constant. This is the welfare-theoretically correct loss function for the central bank to minimize in order to maximize the household's utility.

Once we make an assumption about how the central bank uses the objective function (4) to set the path of interest rates $\{i_t\}$, then equations (1) through (4) fully specify the model.

Note that I choose to write the entire model in terms of the output gap $x_t$, rather than in terms of the actual level of output itself, unlike many papers on optimal policy at the ZLB. While, of course, writing the model the two ways is completely equivalent, it will turn out that writing the model in terms of the output gap is quite valuable for generating sharp intuition for the results. This is because the output gap is directly the *welfare-relevant* object from the loss function – unlike output or consumption – and therefore makes discussion of *optimal*, welfare-maximizing policy especially clear.

I transform the infinite-horizon model to a simplified two-period model by assuming that: from period 2 onwards, all prices are completely flexible – so the output gap is forever zero – and

[3] I assume throughout that the fiscal authority implements the usual production subsidy to offset the distortion from market power, and that it offsets and time variation in the wedge between flexible-price output and first-best output so that there are no shocks to the NKPC (i.e. so-called "cost push" shocks).

[4] In particular, this assumes an absence of productivity shocks. Section 5 incorporates productivity shocks.

assume that the central bank then chooses to set inflation to zero.[5] As a result, the model collapses to a small system of equations:

1. The NKPC in period 0 and in period 1:
$$\pi_0 = kx_0 + \beta E_0\pi_1 \tag{5}$$
$$\pi_1 = kx_1 + \beta E_1\pi_2$$
$$= kx_1 \tag{6}$$
2. The EE in period 0 and in period 1:
$$x_0 = E_0x_1 - \sigma[i_0 - E_0\pi_1 - r_0^n] \tag{7}$$
$$x_1 = E_1x_2 - \sigma[i_1 - E_1\pi_2 - r_1^n]$$
$$= -\sigma[i_1 - r_1^n] \tag{8}$$
3. The welfare loss function of the central bank:
$$\mathbb{W} = \pi_0^2 + \lambda x_0^2 + \beta[\pi_1^2 + \lambda x_1^2] \tag{9}$$

We assume that, due to the existence of non-interest bearing cash as an outside option, there is a zero lower bound on nominal interest rates.[6]

$$i_0, i_1 \geq 1 \tag{10}$$

This could easily be generalized to an "effective lower bound" at some exogenous level below zero.

Finally, since we want the ZLB to bite in period 0 but not in period 1, we assume that the natural rate equals its steady state value of $1/\beta$ in period 1; but that in period 0 it is equal to some level $\phi < 1$ so that the ZLB constraint is relevant:

$$r_0^n = \phi \tag{11}$$
$$r_1^n = 1/\beta \tag{12}$$

## 2.2 Equilibrium

Substituting in the natural rate terms and working backwards from period 1 to period 0, we can express all four endogenous objects – the output gap and inflation in both periods, $\{x_1, \pi_1, x_0, \pi_0\}$ – in terms of the two exogenous interest rates set by the central bank, $\{i_1, i_0\}$.

I highlight the terms of particular interest. At period 1:

$$x_1 = -\sigma\left[i_1 - \tfrac{1}{\beta}\right] \tag{13}$$
$$\pi_1 = -\sigma k\left[i_1 - \tfrac{1}{\beta}\right] \tag{14}$$

At period 0:

$$x_0 = -\sigma(1 + k\sigma)\left[E_0 i_1 - \tfrac{1}{\beta}\right] - \sigma[i_0 - \phi] \tag{15}$$

[5] The choice of zero inflation for $t \geq 2$ merely simplifies the algebra.

[6] As Koning (2013) highlights, this ZLB constraint can be viewed as an implication of Gresham's Law.

$$\pi_0 = -\sigma k[1 + \sigma k + \beta]\left[E_0 i_1 - \tfrac{1}{\beta}\right] - \sigma k[i_0 - \phi] \tag{16}$$

In period 1, if the policy rate $i_1$ is kept below the natural rate $1/\beta$, then there is inflation ($\pi_1 > 0$) and an economic boom ($x_1 > 0$). In period 0, whether inflation and the output gap are positive or negative depends on the entire path of policy rates $\{i_0, i_1\}$ compared to each period's natural nominal rates $\{\phi, 1/\beta\}$.

Under the welfare loss function (9), the central bank would like to ensure that inflation and output gap are zero in both periods, $\pi_t = x_t = 0$. In the absence of the ZLB, this could easily be done by setting the policy rate in each period equal to the natural rate, $i_0 = \phi$ and $i_1 = 1/\beta$. When $\phi < 1$ so that the ZLB binds in period 0, however, this is not feasible; the policy rate is stuck at $i_0 = 1$.

### 2.3 Central banks are not "pushing on a string" at the ZLB

In popular discourse, it is common to hear claims of total monetary policy impotency under a binding ZLB constraint: "central banks are pushing on a string" in terms of ability to raise inflation, and similar cliched analogies.

It is worth emphasizing that this is simply not the case in our standard textbook models, New Keynesian or otherwise, if the economy is expected to ever have any probability of leaving the ZLB.[7] This point, first made formally in the seminal Krugman (1998) paper, can be seen through the lens of the model here in equations (15) and (16). Supposing the nominal rate in period 0 is constrained by the ZLB so that $i_0 = 1$, then inflation and the output gap during this "liquidity trap" are:

$$x_0 = -\sigma(1 + k\sigma)\left[E_0 i_1 - \tfrac{1}{\beta}\right] - \sigma[1 - \phi] \tag{17}$$

$$\pi_0 = -\sigma k[1 + \sigma k + \beta]\left[E_0 i_1 - \tfrac{1}{\beta}\right] - \sigma k[1 - \phi] \tag{18}$$

Clearly, the level of inflation during the "liquidity trap", $\pi_0$, can be raised by the central bank's choice of *future* policy $i_1$. Thus, the notion of "forward guidance": if the central bank at time 0 can make credible promises about future policy, $i_1$, they can manipulate inflation during the liquidity trap, $\pi_0$.

Is there any sense, then, in which the ZLB a "problem" for central banks? There are two:

(1) Promises for future policy action may not be believed: forward guidance may be time inconsistent.

[7] For the case when the economy is never expected to have even epsilon probability of leaving the ZLB, see the literature on secular stagnation (e.g. Eggertsson, Summers, and Mehrotra 2016).

(2) The "fewer-instruments-than-targets" problem: although the central bank can affect inflation during the liquidity trap $\pi_0$, the ZLB constraint means that it cannot achieve the first best and set inflation and the output gap to zero in both periods.

I now lay out these two issues, before discussing in sections 3 and 4 the results which show caveats to the importance of each.

### 2.4 The "expectations trap": the time consistency of monetary policy at the ZLB

Consider first a central bank which is unable to credibly commit to its promises and operates under discretion. Such a central bank minimizes the welfare loss function period-by-period. In period 1 after the ZLB episode has ended, the central bank seeks to set the interest rate $i_1$ to minimize the welfare loss, subject to the equilibrium conditions (7) and (8):[8]

$$\min_{i_1} \pi_1^2 + \lambda x_1^2 \tag{19}$$

$$\text{s.t.} \quad \pi_1 = -\sigma k\left[i_1 - \tfrac{1}{\beta}\right]$$

$$x_1 = -\sigma\left[i_1 - \tfrac{1}{\beta}\right]$$

Observe that the central bank can perfectly minimize its loss function in period 1 by setting the nominal rate equal to the natural rate $r_1^n = 1/\beta$:

$$i_1 = \tfrac{1}{\beta}$$

By setting the nominal rate equal to the natural rate, this ensures zero inflation, $\pi_1 = 0$, which ensures that the underlying Calvo price rigidity has no consequence for real output or the output gap, $x_1 = 0$.

Note that this optimal policy problem and its implication – "set the nominal rate equal to the natural rate" – is entirely independent of the ZLB constraint. That is, whether or not the ZLB were to bind at time 0, this would be optimal policy at time 1 for a discretionary central bank.

Now compare this to optimal policy as set by a central bank which is able to credibly commit to its promises. Such a central bank maximizes lifetime welfare (9), by choosing the interest rate *path* $\{i_0, i_1\}$, subject to the ZLB constraint (10) and the equilibrium conditions (13)-(16):

$$\max_{i_0, i_1} \pi_0^2 + \lambda x_0^2 + \beta[\pi_1^2 + \lambda x_1^2] \tag{20}$$

$$\text{s.t.} \quad i_0 \geq 1$$

and (13)-(16)

One can then substitute out the endogenous objects $\{\pi_t, x_t, r_t^n\}$ using the equations (13)-(16) derived above and take the first order conditions on the central bank's problem under commitment. After tedious algebra, and again under assumption (7) that the ZLB binds at $t = 0$, optimal commitment policy is derived as:

[8] The ZLB does not bind at time 1, so for clarity I ignore it here.

$$i_0 = 1$$
$$i_1^* = \frac{1}{\beta} - \alpha(1-\phi) \tag{21}$$

Here, $\alpha$ is an unimportant positive constant.[9]

What is the interpretation? In period zero, the central bank sets the nominal rate as low as possible, at the zero lower bound. In period one, the central bank commits itself to follow through on *expansionary forward guidance*: the optimal interest rate after the ZLB under commitment, $i_1^*$, is *below* the natural rate $r_1^n = 1/\beta$ since $1-\phi > 0$ by assumption and $\alpha > 0$. The more binding is the ZLB during the liquidity trap, then the stronger is the forward guidance, i.e. the smaller is $\phi$ the smaller is $i_1^*$. Observe from equations (13) and (14) that this policy of setting $i_1 < r_1^n = 1/\beta$ results in positive inflation and a boom in the output gap during period 1.

Thus, under optimal commitment policy, the central bank promises to have a lower interest rate in the future than would otherwise be optimal without the ZLB, which creates a boom in period 1 after the ZLB episode. Expectations during the liquidity trap at $t = 0$ for that future boom at $t = 1$ help to ameliorate the deflationary recession during the liquidity trap.

Finally, we can observe the time consistency problem of the ZLB: optimal policy under commitment differs from optimal policy under discretion. The discretionary central bank, unable to keep its promises, would like to promise to keep the nominal rate low after the ZLB episode ends; but it is unable to commit to follow through on its forward guidance, and it reverts to setting $i_1 = 1/\beta$ and does not create the post-ZLB boom. By definition of the max operator, such a discretionary policy achieves lower lifetime utility: the world would be better off if the central bank could commit. Hence, the idea that there is an "expectations trap" at the ZLB due to the time consistency problem, rather than a "liquidity trap" where the central bank is mechanically unable to affect inflation (Krugman 1998; Eggertsson and Woodford 2003).

### 2.5 The fewer-instruments-than-targets problem

The expectations trap is the first challenge created by the ZLB, but there is another: even if the central bank can commit, it still cannot achieve the first best. In the first-best world, the central bank would be unconstrained by the ZLB, and would be able to set the policy interest rate equal to the natural rate in both periods – $i_0 = \phi$ and $i_1 = 1/\beta$ – which would ensure both zero inflation and zero output gap in both periods, $\pi_t = x_t = 0$, which would be optimal given the objective function (9).

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[9] $\alpha = \frac{B}{A} > 0$, where $B \equiv k^2[1 + \sigma k + \beta] + \lambda(1 + k\sigma) > 0$ and $A \equiv k^2(1 + \sigma k + \beta)^2 + \lambda(1 + k\sigma)^2 + \beta k^2 + \lambda\beta > 0$. We also clearly need to assume a configuration of parameters such that the ZLB constraint does not bind at $t = 1$, i.e. $i_1^* \geq 1$.

The reason the central bank cannot achieve the first best at the ZLB, even under commitment, is that the ZLB constraint means that there are in effect fewer instruments than targets, a la Poole (1970). In effect, there are two targets – zero inflation in each period, $\pi_0 = 0$ and $\pi_1 = 0$, since zero inflation would also ensure zero output gap – but only one free instrument $i_1$, because the instrument at time 0 is locked by the ZLB, $i_0 = 1$. As a result, there is the optimal policy rate $i_1^* < r_1^n$ derived above, which trades off: the benefit of bringing inflation during the ZLB, $\pi_0$, closer to the desired level of zero *versus* the cost of raising inflation afterwards, $\pi_1$, above zero.

As an aside to the main framework used in this paper, I note that while this second "fewer-instruments-than-targets" problem arises in the mainline New Keynesian mode, it does not appear in all microfounded business cycle models. Under the staggered pricing of the Calvo (1983)-Yun (1996) friction used in the New Keynesian framework, any amount of inflation creates inefficient price dispersion, because otherwise-identical firms are forced to make different pricing decisions by the exogenous Calvo fairy. This means that a promise by the central bank during a liquidity trap for inflation after the episode ends necessarily creates distortions. These distortions lower welfare and prevent the central bank from achieving the first best even when it can commit.

Under other, non-Calvo nominal rigidities where pricing decisions are more synchronized, forward guidance is not distortionary, and thus credible monetary policy can achieve the first best.[10] Mankiw and Weinzierl (2011) study optimal monetary and fiscal policy at the ZLB in such an environment, where all prices are completely rigid in period 0 and completely flexible in period 1; see also Auerbach and Obstfeld (2005).[11] One-period information frictions have the same property, such as in the model of Lucas (1972) or in simple versions of the "sticky information" of Mankiw and Reis (2002). The necessary condition for achieving efficiency in these and other models is that *pre-announced* inflation is not distortionary, so forward guidance is not distortionary.

Having set out the baseline ZLB model, I now turn to my five theses, beginning with two on optimal monetary policy relating to the *two* challenges created by the ZLB just described.

# 3 Thesis 1: the time consistency problem for monetary policy at the ZLB can be easily overcome by reputational effects

[10]As just one example, in the menu cost model of Caratelli and Halperin (2023), sufficiently strong forward guidance would cause all firms to adjust, and thus would not be distortionary except for the direct welfare loss from menu costs themselves.

[11] The justification for the staggering assumption of the Calvo-Yun formulation and other time-dependent pricing rules is the claim that price dispersion is present in the data; see Taylor (2016, sec. 3) for a review.

As is well-known in game theory, repeated games are fundamentally different from one-shot games. For instance, although the prisoner's dilemma is truly a dilemma if played once, in a repeated game, cooperation has the possibility of being sustained.

In this section, I extend the baseline ZLB model developed above from a one-shot game where the ZLB binds only once, to a repeated game where there is always a chance that the ZLB may bind again in the future. I show that if the frequency of future ZLB episodes is high enough, then the desire by a discretionary central bank to renege on its forward guidance is outweighed by its desire to maintain its reputation for future episodes, and there is no time consistency problem.

Nakata (2018) has previously highlighted this point explicitly in a larger-scale model, with calibration to data. I also discuss how this result is analogous to the way that Barro and Gordon (1983) showed that desire for an inflation-fighting reputation can overcome the inflationary time consistency problem that central banks face in the model of Kydland and Prescott (1976).[12]

### 3.1 A model of ZLB cycles

Consider an infinite-horizon model, where each period consists of two sub-periods. Those two sub-periods map to periods $t = 0$ and $t = 1$ of the baseline model; there is no connection between sub-periods of one period with sub-periods of a separate period. Thus, the model consists of playing the baseline model, repeatedly. We can conceive of a single period as a "business cycle", where subperiod 0 is the potential ZLB event and subperiod 1 is the successive recovery until the next business cycle, with connections between business cycles severed for the sake of clarity.

For a given variable $X$, denote $X_{t(0)}$ the value of the variable at time $t$ in subperiod 0, and $X_{t(1)}$ likewise for subperiod 1.

It is now interesting to allow the natural rate during the ZLB to be stochastic. Suppose with probability $p$ the ZLB binds in subperiod 0; otherwise the natural rate is at its steady-state level and the ZLB does not bind.

$$r^n_{t(0)} = \begin{Bmatrix} \phi & \Pr = p \\ 1/\beta & \Pr = 1 - p \end{Bmatrix} \tag{22}$$

In subperiod 1, the natural rate is always equal to the steady-state level, $r^n_{t(1)} = 1/\beta$, as in the baseline model.

The central bank's lifetime loss function is:

$$\mathbb{W} = \sum_{t=0}^{\infty} \delta^t \left[\pi^2_{t(0)} + \lambda x^2_{t(0)} + \beta\left[\pi^2_{t(1)} + \lambda x^2_{t(1)}\right]\right] \tag{23}$$

[12] Stokey (1989, 1991) generalizes this point to other seemingly time inconsistent government policies.

$\beta$ is now the discount rate between *sub*periods, whereas $\delta$ is the (potentially different) discount rate between periods. By differentially setting these discount factors, we effectively allow for the two subperiods to be of different lengths: recessions and recoveries may be of different duration.

Exactly as in the baseline model of section 2, we can solve for the four equilibrium objects of each period $\{\pi_{t(0)}, x_{t(0)}, \pi_{t(1)}, x_{t(1)}\}$. These equations have exactly the same form as equations (13)-(16), except merely for the notational difference to account for the existence of subperiods.

$$x_{t(1)} = -\sigma\left[i_{t(1)} - \tfrac{1}{\beta}\right] \tag{24}$$

$$\pi_{t(1)} = -\sigma k\left[i_{t(1)} - \tfrac{1}{\beta}\right] \tag{25}$$

$$x_{t(0)} = -\sigma(1+k\sigma)\left[E_{t(0)}i_{t(1)} - \tfrac{1}{\beta}\right] - \sigma\left[i_{t(0)} - \phi\right] \tag{26}$$

$$\pi_{t(0)} = -\sigma k[1+\sigma k+\beta]\left[E_{t(0)}i_{t(1)} - \tfrac{1}{\beta}\right] - \sigma k\left[i_{t(0)} - \phi\right] \tag{27}$$

where $E_{t(0)}$ denotes expectations made at time $t$ in subperiod 0.

### 3.2 Optimal policy under discretion, without reputation

As a baseline, consider the case where the central bank is myopically operating under discretion, without considerations for reputation. Optimal policy under discretion then is exactly the same as in the baseline model with discretion. In subperiod 0 of every period, the central bank sets the nominal rate equal to the natural rate if possible, otherwise to zero; and in subperiod 1 sets it equal to the natural rate.

$$i_{t(0)} = \begin{Bmatrix} 1 & \text{if } r^n_{t(0)} < 1 \\ r^n_{t(0)} & \text{else} \end{Bmatrix}$$

$$i_{t(1)} = 1/\beta$$

If the ZLB binds in subperiod 0, there is a recession and deflation.

$$x_{t(0)} = \begin{Bmatrix} -\sigma(1-\phi) < 0 & \text{if } r^n_{t(0)} < 1 \\ 0 & \text{else} \end{Bmatrix}$$

$$\pi_{t(0)} = \begin{Bmatrix} -\sigma k(1-\phi) < 0 & \text{if } r^n_{t(0)} < 1 \\ 0 & \text{else} \end{Bmatrix}$$

In subperiod 1, optimality is achieved, with the output and inflation at zero.

$$x_{t(1)} = 0$$

$$\pi_{t(1)} = 0$$

As discussed in section 2, the central bank could achieve higher welfare in periods where the ZLB binds if it were able to commit to a lower future interest rate, $i_{t(1)} < 1/\beta$, in order to ameliorate the recession and deflation in subperiod 0.

### 3.3 Optimal policy under commitment

Now consider the case where the central bank can operate under commitment.

The optimal commitment problem involves choosing a state-contingent plan for interest rates, depending on if the natural rate in subperiod 0 realizes as $r^n_{t(0)} = \phi$ and the ZLB binds, or if not and $r^n_{t(0)} = 1/\beta$. Denote these as states $s_{t(0)} \in \{L, H\}$ where the low state $s_{t(0)} = L$ is the case where the ZLB binds and $r^n_{t(0)} = \phi < 1$; and the high state $s_{t(0)} = H$ otherwise. Denote the interest rate in subperiod 0 of time $t$ in each state $s$ as $i_{t(0)}(s)$ and in subperiod 1 as $i_{t(1)}(s)$.

The optimal commitment problem then is to choose a state-contingent sequence for the interest rate to minimize the expected lifetime loss function (14):

$$\min_{\{i_{t(0)}(s),\ i_{t(1)}(s)\}_{s\in\{L,H\}}} \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t \left[\pi^2_{t(0)} + \lambda x^2_{t(0)} + \beta\left[\pi^2_{t(1)} + \lambda x^2_{t(1)}\right]\right]$$
$$\text{s.t. } i_{t(0)}(s), i_{t(1)}(s) \geq 1 \quad \forall s, t$$
$$\text{and (24)-(27)} \quad \forall s, t$$

It is easy to see that optimal policy under commitment policy is, in each cycle, the same as the baseline model, since there is no connection between policy in one cycle and outcomes in another. That is:

1. If the ZLB is binding in subperiod 0 since $r^n_{t(0)} = \phi$, then it is optimal to set the nominal rate in subperiod 0 at the ZLB, $i_{t(0)} = 1$, and to set the nominal rate in subperiod 1 equal to the optimal level from commitment policy in the baseline model, $i_{t(1)} = i^*_1$.
2. If the ZLB is not binding in subperiod 0 since $r^n_{t(0)} = 1/\beta$, then it is optimal to set the nominal rate in both subperiods equal to the natural rate, $i_{t(0)} = i_{t(1)} = 1/\beta$, so that there is neither inflation nor any output gap.

Written formally, optimal policy under commitment is:

$$\begin{aligned} &i_{t(0)}(L) = 1, && i_{t(1)}(L) = i^*_1 \\ &i_{t(0)}(H) = 1/\beta, && i_{t(1)}(H) = 1/\beta \end{aligned} \tag{28}$$

where $i^*_1$ was defined in equation (21).

### 3.4 Optimal policy under discretion, with reputation

So far we have considered: optimal policy under discretion without reputation; and optimal policy under commitment.

Now consider the case where the central bank operates under discretion – but suppose that the public plays a one-period punishment when setting their expectations, so that there is a reputational mechanism, which I will show to be rational. In particular, suppose that the public expects during a ZLB episode today that:

1. If during the last business cycle the central bank did *not* follow through on its forward guidance, then it will again not do so during this business cycle.
2. If, on the other hand, during the last business cycle the central bank did not deviate from any promises, then it will keep any promise made during this business cycle.

In math, this expectations rule is written as:

$$\mathbb{E}_{t(0)}\left[i_{t(1)}(L)\right] = \left\{\begin{matrix} i_t^* & \text{if} \quad i_{t-1,(1)} = E_{t-1,(0)}\left[i_{t-1,(1)}\right] \\ \frac{1}{\beta} & \text{else} \end{matrix}\right\} \tag{29}$$

This brings us to proposition 1, the main result of this section.

**Proposition 1:** If the probability of a ZLB episode $p$ is above a threshold $\overline{p}$, then a discretionary central bank playing the optimal commitment policy (28) together with the public playing one-period punishment (29) is an equilibrium. That threshold is:

$$\overline{p} \equiv \frac{\beta}{\delta}\left(1 + \sigma k + \beta \frac{[k^2[1+\sigma k+\beta]+\lambda+k^2]}{[k^2[1+\sigma k+\beta]+\lambda+k\sigma\lambda]}\right)^{-2} \tag{30}$$

Restated, theorem 1 says that if ZLB episodes are sufficiently frequent, then even a central bank which does not have commitment power can nonetheless successfully implement the optimal commitment policy, in the face of the one-period punishment strategy by the public.[13] The intuition here is, as emphasized above, that the central bank trades off the gain from reneging on its promise and ensuring zero inflation, versus the cost of losing its credibility if it faces another ZLB episode in the next period.

The proof reflects that intuition, as the proof is directly from the single-period deviation criterion. Per that criterion, the conjectured equilibrium is indeed an equilibrium if and only if

$$\underbrace{\beta\left(\left[\pi_{t(1)}^{\text{commit}}\right]^2 + \lambda\left[x_{t(1)}^{\text{commit}}\right]^2\right)}_{\text{gain from cheating today}} \tag{31a}$$

$$<$$

$$\underbrace{\delta p\left(\left[\pi_{t+1,(0)}^{\text{disc}}\right]^2 + \lambda\left[x_{t+1,(0)}^{\text{disc}}\right]^2\right) - \delta p\left(\left[\pi_{t(0)}^{\text{commit}}\right]^2 + \beta\lambda\left[x_{t(0)}^{\text{commit}}\right]^2 + \beta\left[\pi_{t(1)}^{\text{commit}}\right]^2 + \beta\lambda\left[x_{t(1)}^{\text{commit}}\right]^2\right)}_{\text{discounted expected loss from punishment tomorrow}} \tag{31b}$$

$\pi^{\text{commit}}, x^{\text{commit}}$ denote outcomes under commitment play in the low state; $\pi^{\text{disc}}, x^{\text{disc}}$ denote outcomes under discretionary play without reputation in the low state, as this is what occurs under punishment.[14] The term in (31a) is the welfare loss in period $t$, in subperiod 1, which is

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[13] An alternative, equivalent framing would be: if the central bank is sufficiently patient, i.e. $\beta$ is sufficiently high, then optimal commitment policy can be sustained.

[14] That is:
$\pi_{t(1)}^{\text{commit}} = \sigma k \alpha (1-\phi)$

avoided by cheating and playing $i_{t(1)} = 1/\beta$. The first term in (31b) is the expected discounted welfare in $t+1$ under punishment when the central bank is forced to act without forward guidance. The second term in (31b) is the expected discounted value of what welfare would have been in $t+1$ if the central bank did not deviate. The chance that the ZLB is binding in $t+1$, captured by the probability parameter $p$, is what creates the chance for punishment. Simplifying the algebra gives the condition (30) of proposition 1.

The result in proposition 1 can be strengthened further if the public plays a multi-period – or permanent grim-trigger – punishment strategy. That is, under these stronger punishments, the threshold probability for ZLB episodes necessary to sustain forward guidance is even lower than that given in theorem 1.

**3.5 Relation to Barro and Gordon (1983) and discussion of post-Great Recession policy**

As noted in the introduction to this section, it is a basic implication of microeconomic game theory that commitments which are noncredible in one-shot games can be credible in repeated games. This is simply an implication of the folk theorem. The math above merely formalizes this in our setting.

The logic of theorem 1 – that reputation can overcome the *deflationary* time consistency problem of the ZLB – is also directly analogous to the work of Barro and Gordon (1983) on how reputation can overcome an *inflationary* time consistency problem facing central banks. Kydland and Prescott (1976) had set up an environment where the central bank is continually tempted to implement inflationary policy to goose the economy; however, the public recognizes this temptation, and merely raises their inflation expectations to account for this. The result is higher inflation, without any of the benefits of stimulative monetary policy. Central banks in this setup thus have an inflationary bias caused by a time consistency problem, where they would like to commit to low inflation, but then would be tempted to deviate to goose the economy.

Barro and Gordon (1983) point out in reply that central banks are playing a repeated game with the public, and concern about reputation can overcome the Kydland-Prescott time consistency problem. Although the central bank may be continually tempted to goose the economy for the sake of monetary stimulus, it also cares that the public trust it in the future not to inflate. That

$x^{\text{commit}}_{t(1)} = \sigma\alpha(1-\phi)$
$\pi^{\text{commit}}_{t(0)} = \sigma k[1+\sigma k+\beta]\alpha(1-\phi) - \sigma k[1-\phi]$
$x^{\text{commit}}_{t(0)} = \sigma(1+k\sigma)\alpha(1-\phi) - \sigma[1-\phi]$
$\pi^{\text{disc}}_{t+1,(0)} = -\sigma k[1-\phi]$
$x^{\text{disc}}_{t+1,(0)} = -\sigma[1-\phi]$

trust is necessary to keep the economy steady in the future. It thus has to trade off the small temporary gain from goosing the economy today with the potentially permanent loss to its reputation, which could make it worse off for the entire future. For small enough temporary gains or large enough punishments, together with a sufficiently low discount rate, low inflation can be sustained as an equilibrium and the inflationary bias overcome.

This time consistency of policy seems to be very much the relevant case empirically. Since the 1980s, developed-world central banks have achieved low inflation. Central banks like the Federal Reserve or European Central Bank spend a substantial portion of their energy on attempting to communicate clearly with the public and continuously stress the importance of maintaining their inflation-fighting reputations.

I do not here attempt to do a full quantitative analysis of whether the Federal Reserve or European Central Bank have implemented the commitment-optimal policy during the multiple ZLB episodes of the last 15 years. Speaking qualitatively, in the recovery from the Great Recession, there seems not to have been any period of above-target inflation or economic boom, which is what such policy would prescribe.

The seeming failure to implement optimal commitment policy was despite an overwhelming body of research during the period arguing that the ZLB would be highly likely to bind in years to come (e.g. Kiley and Roberts 2017) as well as explicit acknowledgment of such probabilities by monetary policymakers, such as Jerome Powell.[15] This is suggestive of an important policy failure during the aftermath of the Great Recession. Reiterating my paraphrase of the Krugman (1998) slogan from the introduction: it should be easy – for a central bank which properly has concern about its reputation – to credibly promise to be 'irresponsible' at the ZLB, since its reputation is on the line.

# 4 Thesis 2: the ZLB is not special

*[to be written up]*

# 5 Thesis 3: austerity is stimulus

## 5.1 Setup

[15] As reported in WSJ (2019), "'The next time policy rates hit the [lower bound] — and there will be a next time — it will not be a surprise,' Mr. Powell said". Nakata and Sunakawa (2018) appendix H also collects quotes from an international set of monetary policymakers expressing concern about their time consistency.

I now turn to three theses on fiscal policy in the representative agent New Keynesian model (RANK). I emphasize that these conclusions are derived in the representative agent version of the New Keynesian framework in order to highlight that Ricardian equivalence holds, so that "fiscal stimulus" is modeled as government consumption (or investment). The recent heterogeneous agent New Keynesian (HANK) literature often analyzes "stimulus" as *transfers* (e.g. Wolf 2021), which in the Ricardian setting of the RANK model analyzed here have no effect.[16] That said, the insights for fiscal purchases analyzed here are still relevant to understanding the effect of fiscal purchases in HANK models.

To analyze optimal fiscal stimulus, I extend the baseline model of section 2 to include government spending on public goods, $g_t$, exactly as in Werning (2012), Woodford (2011), and Eggertsson (2001). I highlight terms related to this government spending in red.

Since I have written the model in terms of the output gap rather than in terms of output itself, neither the NKPC equation (1) nor the EE equation (2) of the baseline model need change their structure.[17,18] The introduction of government spending only changes the path for the natural rate (3) and the welfare loss function (4).

With government spending, the per-period welfare loss function is:[19]

$$\mathbb{W}_t = \pi_t^2 + \lambda x_t^2 + \lambda_g g_t^2 \tag{32}$$

The log-deviation of government spending, $g_t$, enters the welfare loss function quadratically: *either* a decrease *or* an increase in government spending $g_t$ directly reduces welfare. This is an implication of microeconomic efficiency concerns. There is some level of government production of public goods which is optimal, as determined by the canonical Samuelson rule for public goods provision. If the production of public goods is below this level, then public goods are underprovided. But also, symmetrically, if the production of public goods is above this level, then households need to overwork in order to produce these public goods. This overproduction is costly due to the

[16] The conclusion section expands on these points and offers further relevant citations.

[17] The output gap is consistently defined as the difference between log-deviations in output $y_t$ and log-deviations in the flex-price level of output, $y_t^f$: that is, $x_t \equiv y_t - y_t^f$. The introduction of government spending does affect $y_t^f$, the underlying flexible-price level of output. As the appendix shows, $y_t^f = \Gamma g_t$ where $\Gamma > 0$ is a constant discussed in footnote 20 and the appendix. However, the dynamics of the output gap and inflation (equations 1 and 2) are not affected by this change.

[18] I abuse notation, however, since the parameter $\sigma$ now needs to be normalized. Whereas without government spending $\sigma$ was the inverse of the intertemporal elasticity of substitution, now it is this inverse elasticity divided by the steady state ratio of consumption to output. See the appendix for further detail.

[19] This, as before in equation (3), is a second-order approximation around the efficient steady state. In particular, this is taken around the steady state where the fiscal authority has implemented the Samuelson rule for the optimal quantity of public goods. See the appendix for further detail.

disutility of the required excess labor. Hence, the quadratic loss term, weighted by the parameter $\lambda_g > 0$ which reflects the relative importance of this policy goal.

### 5.2 The natural rate under fiscal stimulus

Critically for the next two sections, the natural rate process is also affected by government purchases. In the baseline model equation (3), the natural rate merely reflected time preference: $r_t^n = \rho_t$ where $\rho_t$ would either be the shock of $\phi < 1$ or the steady state level of $1/\beta$.

With government spending, the natural rate becomes:

$$r_t^n = \rho_t - \gamma E_t \Delta g_{t+1} \tag{33}$$

Here, $\gamma \geq 0$ is a parameter detailed in a footnote[20], which is strictly nonzero unless *both* of the following hold:

1. The household has GHH preferences, so that there are no income effects on labor supply; *and*
2. Production technology is constant returns to scale, rather than decreasing returns to scale.

If both of these conditions hold, then fiscal stimulus has *no* effect on the natural rate, inflation, or the output gap in RANK. I will assume this parameter is nonzero, $\gamma > 0$, to ensure that discussing fiscal policy is even interesting in the first place:

**Assumption 1:** Either the household does not have GHH preferences, or production technology is not constant returns to scale, or both.

### 5.3 The baseline NK model with fiscal stimulus

Collecting equations (1), (2), (32), and (33), the baseline New Keynesian model with fiscal stimulus is:

$$\pi_t = kx_t + \beta E_t \pi_{t+1}$$
$$x_t = E_t x_{t+1} - \sigma[i_t - E_t \pi_{t+1} - r_t^n]$$
$$r_t^n = \rho_t - \gamma E_t \Delta g_{t+1}$$
$$\mathbb{W}_t = \pi_t^2 + \lambda x_t^2 + \lambda_g g_t^2$$

Under assumption 1, fiscal stimulus affects the output gap and inflation, and does so exclusively through its effect on the natural rate (33). The interpretation of equation (33) for the mechanism of effect of fiscal stimulus is directly apparent from the math itself:

[20] The parameter $\gamma$ is defined in the appendix: to summarize, $\gamma = \sigma(1 - \Gamma)$. To define $\Gamma$ in turn, suppose that household preferences over consumption $C$ labor $N$ and public goods $G$ are $u(C) - v(N) + H(G)$ and that production technology of any firm $i$ is $Y_i = f(N_i)$. Additionally, define $\tilde{v}(Y) \equiv v\left(f^{-1}(Y)\right)$. Additionally, let $C$ and $Y$ be the steady state levels of consumption and output. Now finally define $\Gamma \equiv \frac{\eta_u}{\eta_u - \eta_{\tilde{v}}}$, and $\eta_u \equiv -\frac{u''}{u'} C$ and $\eta_{\tilde{v}} \equiv \frac{\tilde{v}''}{\tilde{v}'} Y$. See Woodford (2011) for additional discussion.

**Proposition 3:** Only the *change* in fiscal spending $\Delta g_{t+1}$ matters for the output gap $x_t$ and inflation $\pi_t$, not the *level* of fiscal spending.

This equation (24) for the natural rate is not intrinsically new, but the statement of theorem 3 – that *only* the change in spending matters from a welfare perspective – is framed especially clearly by formulating the model in terms of the output gap, rather than in terms of consumption or output.

Raising the level of government spending today, $g_t = g > 0$, while leaving government spending unchanged tomorrow, $g_{t+1} = 0$, is indeed stimulatory. In this case the change in government spending is negative, $\Delta g_{t+1} = -g < 0$, so that the natural rate today $r_t^n$ is pushed up by the amount $\gamma \cdot g$, as can be read off of equation (24). If the *nominal* interest rate is fixed by the ZLB, $i_t = 1$, then this raising of the natural rate is expansionary, since the interest rate gap $i_t - r_t^n$ falls.

But note that it would be *just* as stimulatory to promise to cut future spending by an equivalent amount, $g_{t+1} = -g$, and not raise spending today, $g_t = 0$. This would have the same effect on the *change* in spending, $\Delta g_{t+1} = -g < 0$, and therefore the same effect on the natural rate.[21]

Thus, the titular claim of this section: austerity is stimulus. Raising spending today, $g_t \uparrow$, is *just* as stimulatory as promising to cut spending tomorrow, $g_{t+1} \downarrow$.

### 5.3 Intuition: consumption smoothing

The result that fiscal stimulus in RANK only depends on the change in government spending, rather than the level, is counterintuitive if one thinks of the New Keynesian model as formalizing the Old Keynesian logic for fiscal stimulus, where such spending *mechanically* boosts the economy, without any intertemporal mechanism. What is the intuition for this result?

The key intuition comes from consumption smoothing (Rowe 2012). Consider the aggregate resource constraint (without log-linearization), $Y = C + G$. If assumption 1 does *not* hold, then it can be shown that an increase in government spending $G$ increases output $Y$ one-for-one, leaving consumption $C$ unaffected as a result.[22] Supposing instead (and realistically) that assumption 1 does hold, then an increase in government spending $G$ increases output $Y$ – but strictly less than one-for-one. As a result, consumption $C$ must fall.

[21] Since fiscal spending must return to steady state, this will require (say) $g_{t+2} = 0$, and therefore $r_{t+1}^n$ would be changed by $-\gamma g$. If, as in the baseline model, the ZLB does not bind at $t+1$, then this can easily be offset by monetary policy, and there is precisely zero effect on the output gap and inflation at $t+2$.

[22] Woodford (2011) has a very clear exposition.

Now suppose the representative agent knows that government spending tomorrow is going to be lower than it is today, holding all else constant. From the resource constraint (under assumption 1), this implies that *consumption* tomorrow will be *higher* than it is today: more of total output $Y$ is available for private consumption $C$.

With the knowledge that government spending tomorrow will be lower than today and that therefore *consumption* will be *higher* than today, the representative agent would like to save less today in order to consume more today, to better smooth consumption over time. This reduced desire to save pushes up the underlying natural rate.

**5.4 Further intuition: positive fiscal stimulus is isomorphic to a negative productivity shock**

A further intuition pump to understand fiscal policy in the New Keynesian model is to compare the effect of government spending to the effect of productivity shocks. So far, we have assumed that productivity is constant. With time-varying productivity $a_t$, neither the NKPC (1) nor the Euler equation (2) are affected, but the formula for the natural rate becomes:

$$r_t^n = \rho_t - \gamma E_t \Delta g_{t+1} + \psi \Delta a_{t+1} \tag{34}$$

Here, I have highlighted the new terms introduced by productivity shocks in blue; $\psi > 0$ is a constant parameter.[23]

A temporary negative productivity shock means that productivity is lower today than tomorrow, $a_t < a_{t+1}$, and so $\Delta a_{t+1} > 0$. Meanwhile, positive fiscal stimulus means that government spending is higher today than tomorrow, $g_t > g_{t+1}$, and so $\Delta g_{t+1} < 0$. Paying careful attention to the signs of these various inequalities, we can see in equation (34) that a negative productivity shock and a positive fiscal stimulus have the same sign of an effect on the natural rate. Since neither type of shock directly affects inflation or the output gap in *any* other equation of the model, the two kinds of shocks have the exact same effect on the equilibrium output gap and inflation of the model.[24]

Meanwhile, it is well-appreciated that negative productivity shocks at the ZLB are expansionary. Among many other papers, Eggertsson (2012) discusses this mechanism in theory. Wieland (2019)

[23] As in footnote 15, the definition of flex-price output $y_t^f$ changes once we incorporate time-varying productivity, which feeds into the output gap $x_t = y_t - y_t^f$ where $y_t$ is (the log deviation of) output. With time-varying productivity, $y_t^f \equiv \Gamma g_t + \psi \sigma^{-1} a_t$, where again $\psi$ is a positive constant defined in the appendix.

[24] Positive government spending shocks and negative productivity shocks have the same effect on the output gap and inflation; but negative productivity shocks have an additional, direct, policy-independent negative effect on welfare. In other words, negative productivity shocks additionally directly lower welfare, in such a way that neither fiscal nor monetary policy can affect. This is swept into the typical "terms independent of policy (t.i.p.)" term of the second-order welfare approximation.

offers a critical empirical evaluation; he looks at the effects of the Great East Japan Earthquake and oil supply shocks and finds them not to be expansionary.[25]

The analogy between fiscal stimulus and negative productivity shocks also provides a less flattering view of what fiscal stimulus is, conceptually, in RANK. Government spending in effect *is* a negative productivity shock here, by making the economy less efficient at transforming labor inputs into household welfare. Both government spending and negative productivity shocks shift inward the utility possibilities frontier.

# 6 Thesis 4: heterogeneous fiscal stimulus can be contractionary

*[to be written up]*

# 7 Thesis 5: fiscal policy is time inconsistent at the ZLB

While the modern liquidity trap literature began with Krugman's (1998) analysis of the time consistency of monetary policy at the zero lower bound, existing work has not characterized the time consistency of *fiscal policy* at the zero lower bound.[26] Eggertsson and Woodford (2003) analyze optimal monetary policy under discretion versus commitment at the ZLB without fiscal policy; Werning (2012) analyzes optimal fiscal policy under commitment, except in the special case where fiscal policy has no stimulative effect. As far as I am aware, no paper has characterized the time consistency of optimal fiscal policy at the ZLB.

Optimal fiscal policy is time inconsistent whether monetary policy operates under commitment or discretion. Because it is simpler and more intuitive, I work with the case where monetary policy operates under discretion.[27]

We now solve for optimal fiscal policy under commitment, and then do so under discretion. We can then compare the two to demonstrate the time inconsistency.

## 7.1 Optimal fiscal policy under commitment

We want to first consider the fiscal authority's problem under commitment, in our baseline two-period environment with the addition of fiscal spending. In line with the assumptions of the

[25] Wieland (2019) argues that both the earthquake and the identified oil supply shocks were temporary, rather than permanent, shocks, which is necessary for the logic here to apply.

[26] See also Sumner (1993), who emphasized the importance of distinguishing between temporary and permanent changes in the money supply in explaining the lack of inflation during a period of American colonial history.

[27] I also assume that fiscal policy and monetary policy are playing a Nash game (rather than e.g. Stackelberg). The time consistency of fiscal policy does not depend on this.

baseline setting, we assume that after period 1, government spending is at its steady state level, $g_t = 0 \;\; \forall t \geq 2$.

As stated above, we will take monetary policy to be operating under discretion. As a result, in period 1 when the ZLB does not bind, the discretionary central bank will always set the nominal interest rate equal to the natural rate; this ensures zero inflation and zero output gap, thereby maximizing its objective function (a). That natural rate is, from equation (33):

$$\begin{aligned} i_1 &= r_1^n = \tfrac{1}{\beta} - \gamma(0 - g_1) \\ &= \tfrac{1}{\beta} + \gamma g_1 \end{aligned}$$

That is, the central bank will always engage in full "monetary offset", offsetting the effect of any fiscal spending $g_1$ on inflation and the output gap.[28] As we saw in section 2.4, this will ensure that period-1 inflation and the output gap are always zero, $\pi_1 = x_1 = 0$.

The fact that period-1 inflation and the output gap are zero considerably simplifies the fiscal authority's optimization problem, since it removes $\pi_1$ and $x_1$ from consideration. Under commitment, the objective is to minimize

$$\begin{aligned} \mathbb{W} &= [\pi_0^2 + \lambda x_0^2 + \lambda_g g_0^2] + \beta[\pi_1^2 + \lambda x_1^2 + \lambda_g g_1^2] \\ &= [\pi_0^2 + \lambda x_0^2 + \lambda_g g_0^2] + \beta \lambda_g g_1^2 \end{aligned}$$

The full optimization problem under commitment is to maximize this objective, subject to the EE and NKPC of period 0:

$$\max_{g_0, g_1} [\pi_0^2 + \lambda x_0^2 + \lambda_g g_0^2] + \beta \lambda_g g_1^2 \tag{34}$$

$$\text{s.t. } x_0 = -\sigma\big[1 - \phi + \gamma E_0[g_1 - g_0]\big] \tag{35}$$

$$\pi_0 = k x_0 \tag{36}$$

Here, the constraints (35)-(36) are the same as equations (15) and (16) after having substituted in the binding ZLB constraint in period 0, $i_0 = 1$, and including the effect of government spending on the natural rate from equation (33).

Examining (34)-(36), we can see that optimal fiscal policy trades off two goals.

1. From equation (35), fiscal policy would like to raise the period-0 natural rate, $r_0 = \phi - \gamma E_0[g_1 - g_0]$, up to the level of the nominal rate of $i_0 = 1$. This would cause the ZLB to not bind, allowing monetary policy to ensure zero output gap and inflation in period 0, which is desirable.
2. On the other hand, from the objective (34), the fiscal authority would like to keep government spending close to its optimal level, to minimize deviations from the Samuelson rule for public goods provision, $g_0 = g_1 = 0$.

[28] As long as the level of government spending is not so negative as to force the central bank back to the ZLB, i.e. $g_1 > -\frac{1}{\gamma}\left(\frac{1}{\beta} - 1\right)$. For more conceptual discussion of the idea of monetary offset, see Sumner (2021).

It is not possible to satisfy both of these goals simultaneously, and optimal policy must trade them off. Lemma 1 solves for the optimal path of spending directly from the first order conditions of the linear-quadratic program (34)-(36).

**Lemma 1.** If fiscal policy can commit and the central bank operates under discretion, then optimal fiscal policy is:

$$g_0^* = \xi[1-\phi] \tag{37}$$

$$g_1^* = -\frac{1}{\beta} \cdot \xi[1-\phi] \tag{38}$$

Where $\xi > 0$ is a constant defined in the appendix.

The interpretation of lemma 1, i.e. optimal fiscal policy under commitment, is that optimally there should be stimulus during the liquidity trap, $g_0^* > 0$, and austerity during the recovery $g_1^* < 0$. This maximizes the impact of fiscal policy on the natural rate in equation (35), while simultaneously smoothing the distortions to the level of government spending in the convex loss function (26). Indeed with no time discounting ($\beta = 1$) then optimally there is perfect smoothing, $g_1^* = -g_0^*$: the level of stimulus during the liquidity trap is the same as the level of austerity during the recovery.

This matches the Old Keynesian logic of "austerity during the boom, stimulus during the bust" but with a very different motive from the traditional logic. The motive here is about smoothing distortions to public goods provision – together with a desire to create a negative productivity shock, which is desirable at the ZLB.

### 7.2 Optimal fiscal policy under discretion

To see the time consistency problem facing fiscal policy, now consider the fiscal authority's problem in period 1 if it were to reoptimize. The period-1 loss function is:

$$\mathbb{W}_1 = \pi_1^2 + \lambda x_1^2 + \lambda_g g_1^2$$

This can again be simplified since the inflation ($\pi_1$) and output gap ($x_1$) terms drop out: because, as above, the discretionary central bank offsets any effect on inflation and the output gap in order to peg them at zero. The period-1 loss function becomes simply:

$$\mathbb{W}_1 = \lambda_g g_1^2 \tag{39}$$

Since objective function (39) is unconstrained, the discretionarily-optimal action at $t = 1$ is simply to set fiscal spending at the steady state level, $g_1 = 0$. Compared to the commitment level of $g_1^* < 0$, the discretionary fiscal authority is spending too high. That is, although the discretionary fiscal authority would ex ante like to commit to austerity in period 1, when that time rolls around, it does not want to engage in the promised austerity.

### 7.3 The time consistency problem for fiscal policy

Proposition 5 summarizes the above discussion.

**Proposition 5.** Optimal fiscal policy at the ZLB is not time consistent. Under commitment, the fiscal authority would like to commit to stimulus $g_0 = g_0^* > 0$ during the liquidity trap and austerity $g_1 = g_1^* < 0$ afterwards. However, without commitment power, it will renege on this promise and set $g_1 = 0$.

Even under discretion, the fiscal authority should still engage in stimulus during the liquidity trap; in fact, it is easy to show that under discretion it should engage in a level of stimulus $g_0^{\text{disc}}$ even higher than what is optimal commitment, $g_0^{\text{disc}} > g_0^* > 0$. This is because knowing that it is not able to raise the natural rate $r_0^n = \phi - \gamma E_0[g_1 - g_0]$ by lowering $g_1$, since it will be unable to follow through on a promise for austerity, it is better off distorting spending in period 0, $g_0$, even more than under commitment in order to raise $r_0^n$ somewhat more to reduce deflation and the output gap.

We have seen that a fully beneficent, aligned fiscal authority would ideally like to commit to stimulus during a liquidity trap and austerity afterwards, but has a motive to renege on its promised austerity. This time inconsistency problem could be worsened if there are political economy motives for fiscal decisionmakers to avoid austerity. Although I do not model it formally here, if reducing fiscal spending harms legislators' reelection prospects, then this could further exacerbate the time inconsistency problem. Of course, just as reputation in a repeated game can help central banks overcome their apparent time inconsistency problem at the ZLB, so can such a reputation mechanism help fiscal policymakers.[29]

## 8 Conclusion

I summarize and reiterate the theses of this paper for optimal monetary and fiscal policy at the zero lower bound in the representative agent New Keynesian model.

1. Monetary policy is not pushing on a string at the ZLB because of the power of forward guidance and expectations; and the force of the time consistency problem in blunting this power is significantly lessened by the fact that central banks want to maintain their credibility and reputation.

[29] Although, quite plausibly, because central banks exist as an institution which carries a reputation, it may be easier for central banks to overcome the time inconsistency problem compared to the fiscal authority, which typically is thought of as carrying less of an institutional reputation but instead varying with the politicians in office.

2. The ZLB is simply a nominal rigidity on one particular relative price. Any nominal rigidity on *any* relative price limits monetary policy in qualitatively the same way: the ZLB is not special.

3. In RANK, fiscal policy affects inflation and the output gap through the *change* in government spending, not the level. This is because positive fiscal stimulus acts as a negative supply shock.

4. In models with heterogeneous goods, positive fiscal stimulus targeted at specific sectors can be contractionary.

5. Optimal fiscal policy at the ZLB is time inconsistent, because the fiscal authority will wish to renege on a promise for austerity after the liquidity trap.

I have discussed these points through a series of small models, meant to serve as intuition pumps for the economic logic – rather than as a quantitatively-accurate model that can be taken to the data.

An important and obvious restriction on the generalizability of the results on fiscal policy is that RANK is a Ricardian model, where the power of fiscal policy is fairly limited. For example, stimulus checks – an important component of policy internationally during the COVID-19 recession – have no impact in the RANK framework (see e.g. Wolf 2021 for an analysis). There is also no distinction in the RANK model between deficit-financed spending versus balanced-budget spending (see e.g. Auclert, Rognlie, and Straub 2018 for an analysis). The burgeoning and active literature on optimal policy in heterogeneous agent New Keynesian models is in the process of analyzing these issues (e.g. Dávila and Schaab 2022; McKay and Wolf 2022; Bilbiie 2021; Acharya, Challe, and Dogra 2023; Bhandari, Evans, Golosov, and Sargent 2021; Le Grand, Martin-Baillon, and Ragot 2021).

Additionally, and as noted in section 2.5, the supply side of the RANK model is based on the canonical Calvo-Yun staggered pricing friction ubiquitous in the New Keynesian literature. Alternative forms of nominal rigidities would have different implications for optimal policy. For example, if nominal rigidities are the result of one-period information frictions as in Lucas (1972) – where imperfect pricing is in a sense synchronized rather than staggered – then forward guidance during a liquidity trap is not distortionary after the ZLB ceases to bind, because *pre-announced* monetary policy actions are not distortionary.[30] Under this nominal rigidity, optimal monetary

[30] For a more general treatment of optimal policy under information frictions, without the ZLB constraint, see Angeletos and La'O (2020) or Iovino, La'O, and Mascarenhas (2021).

policy both can achieve the first best and is time consistent, and fiscal stimulus is unnecessary. The same would hold if prices of all firms were uniformly set one period in advance without the staggering of the Calvo-Yun friction.[31]

[31] Woodford (2003) refers to the resulting equilibrium of the supply block of the model as the "New Classical Phillips Curve".

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